In The Name of Allah
Finite Element
HW # 1
Due 1398.08.25

1. Find the Euler partial differential equation for each of the following functionals:

(a) $\int_a^b \sqrt{1+y'}\,dx$

(b) $\int_a^b \cos(xy')\,dx$

2. Repeat Problem 1 for the following functionals:

(a) $\int_a^b \left(y'^2 + y^2\right) dx$

(b) $\int_a^b \left(5y^2 - y''^2 + 10x\right) dx$

3. Show that a function that minimizes the functional

$$I(\Phi) = \frac{1}{2}\int_S \left[|\nabla\Phi|^2 - k^2\Phi^2 + 2g\Phi\right] dS$$

is the solution to the following differential equation,

$$\nabla^2\Phi + k^2\Phi = g$$

4. Obtain the variational principle for the differential equation

$$-\frac{d^2y}{dx^2} + y = sin\pi x, \quad 0 < x < 1$$

Subject to $y(0) = 0 = y(1)$.